EXAMINATIONS COUNCIL OF ZAMBIA
Examination for School Certificate Ordinary Level.

Additional Mathematics 4030/1
PAPER 1
Friday 28 OCTOBER 2015

Additional materials:
Answer Booklet
Graph paper (1 Sheet)
Mathematical tables, Electronic calculators

Time: 2 hours

Instructions to candidates

Write your name, centre number and candidate number in the spaces on the Answer Booklet provided.

There are 12 questions in this paper. Answer all questions.

Write your answers in the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for candidates

The number of marks is shown in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.
The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones should not be brought in the examination room.
You are reminded of the need for clear presentation in your answers.

Check the formulae overleaf.

This question paper consists of 5 printed pages.
MATHEMATICS FORMULAE

1 ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \).

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Theorem

\[(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)!r!} \)

2 TRIGONOMETRY

Identities

\( \sin^2A + \cos^2A = 1 \)

\( \sec^2A = 1 + \tan^2A \)

\( \csc^2A = 1 + \cot^2A \)

Formulae for \( \Delta ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} bc \sin A
\]
1 Solve the simultaneous equations
\[ x - 2y = 6, \]
\[ x(y - 2) = 12. \] [5]

2 The coordinates of \( P \) and \( Q \) are \((-2, -4)\) and \((6, 4)\) respectively. A perpendicular line to line \( PQ \) passes through \( Q \) and meets the \( y \)-axis at \( R \). Find the coordinates of \( R \). [4]

3 Functions \( f \) and \( g \) are defined by
\[ f: x \rightarrow 1 - 2x, \]
\[ g: x \rightarrow \frac{x}{3 - 4x}, \quad x \neq \frac{3}{4}. \]
Find
(a) \( g^{-1}(x) \), [2]
(b) \( gf(x) \), [2]
(c) the value of \( x \) for which \( gf(x) \) is undefined. [2]

4 Find the range of values of \( k \) for which the line \( y = kx + 3 \) meets the curve \( y = x^2 + 12 \) at two distinct points. [4]

5 (a) In the expansion of \((1 + 3x)^{10}\), the coefficient of \( x^4 \) is \( k \) times the coefficient of \( x^2 \). Find the value of \( k \). [3]
(b) Determine the coefficient of \( x^2 \) in the expansion of \((1 - x + 2x^2)(1 - 3x)^6\). [6]

6 Prove the identity
\[ \cot^2 \theta \sec^2 \theta = 1 + \cot^2 \theta. \] [4]

7 On the same diagram, sketch the graphs of \( y = -3 \cos x \) and \( y = \frac{x}{\pi} \) for the domain \( 0 \leq x \leq 2\pi \). Hence state the number of solutions of the equation \(-3 \pi \cos x = x\) for \( 0 \leq x \leq 2\pi \). [5]

8 The perimeter of a sector of a circle is 16 cm and its area is 16 cm\(^2\). Find
(a) the angle of the sector, [4]
(b) the length of its arc. [3]
9 P, Q and R are points with position vectors \( a - 2b, 4a + b \) and \( 3a - b \) respectively, relative to an origin \( O \). S is a point on \( \overrightarrow{OP} \) produced, such that \( \overrightarrow{OS} = k \overrightarrow{OP} \) and \( \overrightarrow{RS} = m \overrightarrow{RQ} \).

(a) Express \( \overrightarrow{OS} \) in terms of

(i) \( a, b \) and \( k \), [1]

(ii) \( a, b \) and \( m \). [2]

(b) Hence or otherwise, find the values of \( k \) and \( m \). [4]

10 (a) Given that the gradient of \( y = (2x + 3)^n \) at \( x = -1 \) is 4, find the value of \( n \). [3]

(b) Two variables, \( x \) and \( y \) are related by the equation \( y = \frac{2}{(x^2 - 1)} \).

Obtain an expression for \( \frac{dy}{dx} \) and find in terms of \( p \), the approximate change in \( y \) as \( x \) increases from 2 to \( 2 + p \). [6]

11 (a) Find \( \int (5x - 1) \, dx \). [3]

(b) The diagram below shows part of the curve \( y = (4 - x)(x - 1) \) intersecting the \( x \)-axis at \( A \) and \( B \), and meets the \( y \)-axis at \((0, -4)\).

Find

(i) the coordinates of \( A \) and \( B \), [1]

(ii) the area of the shaded region. [6]
12 Answer only one of the following alternatives:

Either

In the diagram below, \( \text{ABE} \) and \( \text{BCE} \) are sectors of circles with centres at \( \text{A} \) and \( \text{B} \) respectively. The radius of sector \( \text{ABE} \) is 8cm and \( \hat{\text{BAD}} = \frac{2\pi}{9} \) radians.

\( \text{ABD} = \text{ARE} = 90^\circ \) and \( \text{ED} = 2.44\text{cm} \).

Calculate the area of the shaded region. \([10]\)

Or

The diagram below shows a quadrilateral \( \text{ABCD} \) in which point \( \text{A} \) is \((0, 6)\) and \( \hat{\text{AEC}} = \hat{\text{ADC}} = 90^\circ \). The equations of lines \( \text{AB} \) and \( \text{AD} \) are \( y - 3x - 6 = 0 \) and \( y + 2x - 6 = 0 \) respectively.

Find

(a) the coordinates of \( \text{B} \) and \( \text{D} \), \([4]\)
(b) the equations of \( \text{BC} \) and \( \text{CD} \), \([4]\)
(c) the coordinates of \( \text{C} \). \([2]\)